THE MATHEMATICS TEACHER

EDITED BY

W. H. METZLER

ASSOCIATED WITH

EUGENE R. SMITH HARRY D. GAYLORD

MAURICE J. BABB WILLIAM E. BRECKENRIDGE

VOLUME V JUNE, 1913 NUMBER 4

EDITORIAL.

It would seem appropriate to call the attention of the members of our association to some obligations that rest upon them. Any organization to be most effective must be thoroughly alive and active. This energy must show in all its activities and permeate all its membership. The officers and members of the Council must be particularly alive and active as it is on this group of members that the business of the association rests. They are the guiding influence of all its activities and should feel keenly the responsibility placed upon them. It is evident, however, that not all members of the Council take this responsibility seriously enough, if the small attendance at some of the meetings is an indication. For the good of our association there should be a marked improvement in interest and attendance on the part of the members of the Council.

Our Association.

Not only must the Council be interested and active but the other members of the association must catch the spirit and be more ready to do their share in the way of taking part and interesting others in becoming members. The territory of the Middle States and Maryland should furnish not less than one thousand members, and our association would soon have that many if every member would interest at least one other to join. In doing this they would be doing both themselves and the new members a service in that every new member makes possible and a more influential organization.

The papers read at the Philadelphia meeting of the Association of Teachers of Mathematics in the Middle States and Maryland, from the standpoint of those not in the teaching profession, throw an interesting light on the condition of mathematics teaching when these men were in school. As long as thoughtful men go on record as believing that the chief value of the study of mathematics is to cultivate the ability of undertaking an uninteresting and disagreeable task, or to show what vocation the boy is not fitted for, we still have something to learn about how to teach.

Point of View

It is unfortunate that teachers so often lack a definite idea of the aim, both of the subject as a whole and of definite parts of it, for, if the teacher is at sea as to his goal, it is little wonder that such ideas as have been quoted come to be believed.

An interesting idea, based on a misunderstanding of correct mathematical teaching, is that of "non-mathematicians" spoken of in one of the papers. In this theory, many of us are judged as totally lacking in mathematical sense, and, therefore, incapable of benefit from its study. I venture to say that any expert teacher of mathematics will support the statement that there are almost, if not quite, no such people as normal "non-mathematicians" in the sense of those incapable of improving their reasoning by its study. That many do not improve their reasoning is unfortunately true, but the fault in such cases is usually with the teaching, not with the subject or its students. Other notable fallacies are the survival of the Puritan idea that to be beneficial the dose must be nauseous, so mathematics made interesting, therefore, fails of its aim, and the statement that a clear thinker on general matters cannot always be taught to think clearly on mathematics. One speaker even asserts that the one thing no one would claim for mathematics is that it teaches good judgment. Most of us believe that if mathematics has one great value to the student who is not to use it in his profession, it is its power to improve judgment and logical ability.

Fortunately, as the teaching of mathematics improves, such mistaken ideas of its aims and results will appear less often.

WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE CURRICULUM OF THE SECONDARY SCHOOLS.*

By Charles L. McKeehan,
of the Philadelphia Bar.

(Continued from last issue.)

I wonder why this is so. However widely opinion may vary as to the value of the study of mathematics as an exercise of mind, surely the great majority of thoughtful persons will agree that arithmetic and algebra and geometry are great and powerful subjects, and that the study of some of these subjects ought to have a place in that limited but vital portion of a boy's education that is supplied by his scholastic training. It would seem too that the kind of training supplied by the study of mathematics is the kind peculiarly needed in youth. Probably few will claim that the elementary branches of mathematics deserve high rank among the studies that train the powers of observation, and develop the inquiring, acquisitive faculty, that cultivate the sense of perspective and discrimination, the power to weigh probabilities. Few would give them high place among the studies that make the mind copious, fertile, vivacious, alert, that cultivate the imagination, elevate the fancy, and enlarge the sympathies. I question whether, if rightly studied, they are of very great value in cultivating a vigorous and retentive memory. But as teaching the power of calm, steady and sustained attention, the acquisition of clear ideas, the habit of accurate and orderly thinking, they surely stand second to no subject now embraced in the curriculum of school or college.

And yet, I believe it to be the fact that nine boys out of ten derive very little useful mental training from the study of mathematics. Whatever benefit they derive from it consists solely, I believe, in a useful training in accuracy and in the wholesome discipline of applying themselves to a tedious and irksome task. Mathematics undoubtedly make for accuracy. Whether a boy scrapes through by an effort of sheer memory or whether he grasps enough of the technique of the subject to

move mechanically through the formulæ to the conclusion (and few boys ever advance beyond one or the other of these methods), still he must be accurate and his conclusions must be correct throughout or his final conclusion will be wrong. And on the other point, it is surely a great gain to a man to learn early in life to set himself to a difficult and irksome task. Certainly he has small chance of success in any of the professions unless he learns sooner or later that day in and day out much of his work is drudgery, but that it must be done promptly, carefully and thoroughly. But for the rest, when you inquire as to the value of mathematics in teaching a boy to think, to reason upon principles and to understand them, to learn from mathematics the real nature and scope of demonstrative proof, the number of boys who derive any appreciable benefit in these lines from the study of mathematics is so small as to be a negligible quantity.

It seems to me that both the *amount* and *kind* of mathematics contained in the present school curriculum is based upon a very exaggerated estimate of a boy's mental capacity. Nothing is more difficult in youth than the acquisition of clear ideas, the power of attention and the habit of orderly logical reasoning. And in mathematics a boy's difficulties are increased tenfold by the abstract and highly technical nature of the subject. He must apply himself to a group of new and difficult conceptions expressed in a highly technical and entirely new and peculiar language. A real grasp of the principles of arithmetic is a task of supreme difficulty for a boy of fourteen years of age. And then too, the technique changes so quickly as he is rushed from one branch of the subject to another. He begins to study history at say eight or nine years of age and continues that subject throughout school and college. He deals throughout with the same language and with the same kind of ideas. But each branch of mathematics has, so to speak, a language all its own. At thirteen or fourteen the student is expected to have *learned* his arithmetic. He has, but he hasn't understood it. And so he is launched into algebra. A new language must be learned, a highly formal technique must be understood before he can begin to contemplate and reason upon and understand the principles of algebra. No doubt the simplest formulæ in algebra contain great principles and lessons and offer rich fields of benefit and

pleasure to those who can and will explore them. But these treasures, like Nature's jewels, are lodged deep in the rocks and the seeker must break and dig and delve to find them. A child of fourteen reads in the introductory chapter of his School Algebra that he is now undertaking "the investigation of the fundamental laws of numbers" and that it is "*very important* that the *beginner* in algebra should have *clear ideas* of these laws and of the extended meaning which it is necessary to give in algebra to certain words and signs used in arithmetic, and that he should see that every such extension of meaning is consistent with the meaning previously attached to the word or sign and with the general laws of numbers." Do you suppose for a single moment that a normal boy of fourteen either can or will do such a thing? And when he tosses that book aside and pulls down Rob Roy or Treasure Island or grabs his hat and makes for the great out-of-doors, where lies the blame? With the boy? Or is it with the teacher who put that book in his hands? A very experienced teacher of mathematics told me not very long ago that in the average freshman class in college not a single student can explain *why it is* that $(-a) \times (-b) = +ab$. Now I have been told that this embodies one of the fundamental conceptions and laws of algebra, and as I find this formula on page 23 of the introductory chapter of my School Algebra, I presume that this is so, and that this is one of the things about which it is "very important that the beginner in algebra should have clear ideas." I venture the statement that the average well-trained, able and successful lawyer can spend a long and arduous evening over the introductory chapter of this School Algebra without getting more than a general idea of what it is about. And if this is true of one of the best school algebras ever written, it must mean that the subject is one of great difficulty, unless you choose to accept Sir William Hamilton's comforting assurance, that "to minds of any talent, mathematics are only difficult because they are too easy."

Would it not be possible to aid the secondary school by some "let up" in the college entrance requirements? Suppose arithmetic only should be required in school and the difficult subjects of algebra and geometry postponed to the maturer years of college. I take it that in a thorough and long-sustained study of

arithmetic between the ages of ten and seventeen, a boy might acquire an understanding of its principles and considerable facility in applying them. He might also, by a long study and use of applied arithmetic, gain a great deal of useful information about numerical relations in the world in which he lives and a truer appreciation of the importance of mathematical values and conceptions. If he subsequently went to college, would he not then be far better fitted for such studies as algebra and geometry? And if he did not go to college, would he not have derived quite as much benefit from the study of mathematics as he now derives during his course at school? He would have traveled a much shorter journey, but would he not have learned the road much better? It is your pedestrian, and not the automobilist, who really *sees* the country. And would he not have gained more in *mental power?* Would he not have seen the difference between learning a thing and understanding it; between *seeming* and *being?* Would he not have gotten a glimmer of what Hobbes meant when he said that "words are the counters of wise men, but the money of fools"? Would he not have learned to plant his feet upon the ground and perhaps grow some day to a realization of Lord Bacon's noble saying that "no pleasure is comparable to the standing upon the vantage ground of truth"

PHILADELPHIA, PA.

MATHEMATICS IN THE SECONDARY SCHOOL, FROM THE POINT OF VIEW OF THE INDUSTRIAL ESTABLISHMENT.

By Sanford A. Moss.

We assume that we have only the case of a young man who is to enter industrial life upon leaving the secondary school and who is not to have a college education. As an introduction it is desirable to enumerate the various directions in which mathematical training received in the secondary school is of benefit to the young man in later industrial life.

The direction of benefit which seems to me by far the most important of all is "vocational directing"; the knowledge that the young man gains regarding his ability to do mathematical work. If he has enjoyed his mathematical courses, which usually means that he has secured good marks, or if the contrary is true, he is given a most important guide in selecting his life's vocation and in planning details of his career. Usually the young man is unconscious of the fact that he is being guided in this way. He has liked mathematical work in high school and unconsciously inclines towards a career which involves mathematics in some way, or he has disliked mathematical work and unconsciously inclines to a career which does not involve it.

"Vocational training" as a means of finding out what a young man is fitted for and starting him straight on his career is now being given considerable well-merited attention. It is usually supposed that this is a matter of trades and manual training. However, secondary school mathematics always has been, perhaps in an unrecognized way, just as much of an aid in vocation selection as a printing school or a manual training wood shop. When this point is fully recognized and when the general subject of vocation selection is as firmly established as it soon bids fair to be, mathematical work will be arranged so as to better secure benefit in this direction.

The direction of benefit which it seems to me should be placed second is the "mind training" in clear thinking given by a successful course in mathematics. One who can do clear thinking

in a mathematical direction can usually do clear thinking in other directions. The reverse does not necessarily hold however. The training in clear thinking which is gained is of use in all other departments of life. This point has often been emphasized and need not be dwelt upon here.

The next direction of benefit I would make "Culture." A mathematical course properly conducted broadens one's outlook and opens up new realms whose existence could not otherwise be imagined. This point also has often been dwelt upon and need not be elaborated here.

It will probably be considered surprising that I place as a direction of benefit less important than the preceding "detailed knowledge of mathematical processes" for use in doing actual work in industrial life. Of course, this is an important direction in which benefit is received but I believe is subordinate to those before mentioned. The young man must have a general knowledge of the fundamental principles of geometry, trigonometry and algebra. However, he will rarely if ever in his later life encounter a problem at all like one which he learned in his text-book. Efforts are usually made to give so-called "practical problems" and there is no doubt that this is wise but no matter how practical we may try to be we can never expect to do more than to inculcate general principles, and leave the details to be worked out as they are met. A high school man in a mechanical drafting-room or a civil engineer's office makes little or no use of that part of his algebra where he was taught long division, least common multiple or such other specific subjects. It rarely happens even that he will be called upon to solve a simple equation, much less a quadratic equation. A young surveyor has little use for methods of solving triangles which he learned in high school. Most of the mathematical processes in industrial life are listed or tabulated or systematized or taken from an engineer's pocket-book in such a way that a young man who has to perform them has merely to follow out set forms which are supplied to him at the time, and has little or nothing to do in the way of exercising his originality. In order to intelligently carry out these set forms he must, of course, have had instruction in the details of the processes involved. However, if he has taken the mathematical courses

and has secured the benefits already enumerated,—the knowledge that he has a mathematical mind, the training in clarity of thinking and the broadening effect of the culture,—he can forget most of the things which he put down on his final examination paper and still do good mathematical work in industrial life.

Do not misunderstand me as saying that because industrial mathematical work is carried out by means of set forms and tables and engineers' pocket books that the training in the elementary processes of the text-books is of no benefit. On the contrary, the set processes alluded to most assuredly cannot be successfully carried out unless the young man has had training in the elements at their foundation.

The fifth and least important direction in which secondary school mathematics is of benefit is in memory training. By this I mean training of that section of the brain which enables us to say things by heart. This is quite a different thing from training the mind in clear thinking previously alluded to. Mind training is training of reasoning faculties and memory training is not. Often this matter of memory thinking is the only benefit which a course in mathematics gives and it is counted as sufficient reason for pursuing the course. I do not agree to this opinion at all. I consider memory training as so much less important than the previous matters that a mathematics course is prostituted if it is principally directed to this feature.

It is obvious that if a young man is to obtain benefit from his mathematical training in the five directions just enumerated, he must have a special aptitude for mathematical work. The discussion of this matter of aptitude which we will now take up I consider the most important point in the whole paper.

Pupils in secondary school mathematical courses can be divided into three classes. First there are those who have so-called mathematical minds and who are at the head of their classes either with little effort or because they enjoy making an effort. The second class pass their subject and to all intents and purposes are doing good work. However, they really know nothing about mathematics as it truly is. They only learn by heart certan rules and can do certain problems only if couched in certain language with which they are familiar. This class includes many girls. Such pupils obtain little or no bene-

fit from high school mathematics as usually taught. The next class of pupils obviously have no talent for mathematical work and do little or nothing. They may be very good pupils and make earnest effort but can accomplish nothing. The last two classes are equally incapable of obtaining mathematical knowledge which will be of benefit to them in industrial life, and must be grouped together as non-mathematicians.

Ability to do mathematical work is just as inherent as is ability to sing in tune or to draw. It is unfortunate that this is not more commonly recognized and instruction for the types of pupils above mentioned arranged accordingly. Take the case of instruction in music or art. No one attempts to make a player out of a person who cannot carry a tune or an artist out of a person who cannot draw. One who can know a tune or can draw is given instruction in all of the technical processes. One who cannot know a tune or cannot draw can nevertheless be given considerable non-technical instruction in music or art. He can be given information regarding the character of the technical processes being used by those who have inherent ability. He can learn of the history of the subject and of the great masters, of the influence on civilization and other similar subjects. No time is wasted in trying to make him hold a pencil or handle a bow. In the case of mathematical work the fundamental aptitude or inaptitude is just as marked and yet we throw all kinds of students together and never seek to separate those who can actually use mathematical processes, and those who cannot use them but could be given instruction in their general nature.

A large majority of the subjects taught in elementary algebra are simply tools of the mathematician's trade. Pupils of the second class above mentioned can apparently learn how to go through the processes involved. However, they can never really use them as tools of the trade. The master violinist may teach his pupil exactly how to hold the bow in his hands and how to hold his fingers in position for the various notes. The pupil may be able to do exactly as he should in these particulars. Yet nothing is accomplished in the way of violin performance unless the pupil has the "divine fire." A boy may be taught to sharpen a wood chisel and to hold it in the proper way in

his hands but he can never be a wood-worker unless he originally possessed an aptitude. So a teacher of algebra may make a pupil multiply trinominals, clear of brackets, and perform the other mechanical operations of algebra, but unless the pupil has a mathematical mind he is simply parroting and can never make any real use of the knowledge he has gained.

In case of grammar school pupils it is necessary that they go through a certain number of things even if they are not fitted for them and do them in parrot way. In the case of a student who is preparing for college a similar situation probably arises although I am by no means certain of this. Still for the sake of argument I am willing to admit that a college student must learn a certain number of things for which he is not adapted in order to intelligently pursue certain other subjects upon which they bear indirectly and for which he may be better adapted. In the case of a student whose school work ceases with the secondary school no such situation arises and there is no reason whatever for giving him instruction in a subject for which he is not fitted. It is no disgrace to a student if he is not fitted for mathematical work. He may be a master in some other direction. Each one of us has a certain direction in which we are best adapted to shine in life and on which we should concentrate most of our energies. The things in this field we can really do. Other things we may learn about in order to broaden our outlook but we should not try to learn to do them, as we never can.

Suppose one had a mixed class of babies and puppies and set about teaching them to walk on two legs. He could teach them all something or other if he put enough time on it. Exactly the same situation exists in any of our classes in algebra. There are some fitted to learn algebra just as well as a baby is fitted to walk on two legs and others fitted just as well as a puppy is to walk on two legs. Furthermore, trying to teach the latter algebra has just as much utility as trying to teach the puppy to walk. The puppy may be a perfectly good puppy and may be able to run and hunt and do all of the other things which a good dog should do but he is just as much out of his sphere when walking on two legs as the baby is at catching a squirrel.

I urge, then, that very early in the secondary school course separation be made of those pupils who can and who cannot

pursue mathematical subjects with profit and that thereafter the mathematicians be not held back by the non-mathematicians and the non-mathematicians be given other subjects in which they can spend their time more profitably. I would like to emphasize a suggestion which I believe has been made often before, that courses be arranged to tell pupils about mathematics who are not fitted to use mathematics. Such courses would include some of the history of mathematics, than which no chapter in history is more interesting or more important. The history of geometry; the story of Euclid, whose original arrangement has needed no essential change to this day; the study of the Hindoo and Egyptian geometers, the part that geometry plays in the marvelous mechanical advances of today and a host of similar subjects will give pupils with non-mathematical minds much more culture and memory training than the present method of trying to make them go through elementary processes in a halting way. There is a great need for a text-book on the "Story of Mathematics" suited for secondary schools and when the masterpiece in this direction is written there will be a great change in our present methods of instruction of non-mathematicians.

We have now enumerated five directions in which a student secures benefit from mathematical training in industrial life: 1st, vocational directing; 2d, mind training; 3d, culture; 4th, detailed mathematical knowledge; 5th, memory training; and have divided the pupils into two classes, the mathematicians and non-mathematicians.

We will conclude by discussing how mathematical instruction should be directed so as to best secure to the two classes of pupils the greatest possible benefit in the five directions.

In the matter of vocational directing the teacher has little to do other than to lead the pupil along to the full extent of his mathematical powers if he is fitted for mathematical work and to lead him to select other subjects if he is not fitted.

In the matter of the second point, mind training, only those who have mathematical minds can secure any benefit no matter how the mathematical teaching is conducted. I believe that no gain in clarity of thought but rather the reverse can come if a non-mathematician pursues a subject in which he is more or less

muddled all the time. The non-mathematicians are therefore wholly barred from training of reasoning powers by mathematical work and there is no use to attempt it.

In the matter of culture the mathematical pupils of course receive a considerable amount from their mathematical work. The presumption is that they enjoy it and are fully awake to all of the processes involved. Therefore, they cannot but help being broadened and being given the advantages which accrue by heart of the mathematical rules even though to the non-mathematicians never really participate in mathematical subjects as they are usually taught and therefore receive no benefit in this direction. If, however, some instruction such as I have suggested in the way of history or story of mathematics can be arranged, non-mathematical pupils could receive considerable culture.

In the matter of the fifth point, memory training, both mathematicians and non-mathematicians receive benefit. Learning by heart of the mathematical rules even though to the non-mathematician they are meaningless trains the memory. However, such memory training for the non-mathematician could just as well be secured in other directions where there will be other benefits besides.

In the matter of the fourth benefit, detailed mathematical knowledge, the non-mathematical pupil can be given as much and the mathematician as little of this as you please, and the latter will soon outstrip the former in any practical use of mathematics. Even in the most routine types of mathematical work in industrial life a non-mathematician, no matter how much he has learned by heart, can never do well, so that training in this direction can never do him any real good.

There remains only to discuss how the subject matter of secondary school mathematics can be arranged to best give detailed mathematical knowledge to the pupils with mathematical minds. As I have already mentioned, most mathematical processes in industrial life are carried on according to certain set forms which are usually learned at the time. No one is called upon to originate any such methods until he has obtained considerable facility in the work by holding subordinate positions for many years. Therefore the mathematical training should be

directed as much as possible towards fundamentals and as little as possible towards details.

To take up specific subjects let us begin with algebra. We of course must include the fundamental processes. However, the manipulative details such as factoring, least common multiple, long division, solution of numerical equations and similar subjects can be omitted entirely or greatly abbreviated. On the contrary the solution of practical problems should be dwelt upon to a large extent. By these, I mean problems which involve setting down of a given verbal description in mathematical form and obtaining an algebraic equation whose solution must be found. I can express my whole idea by stating that I consider as the most important part of this matter the correct setting down of the verbal description in a mathematical form and the least important part the algebraic manipulation of the resulting equation.

In the case of trigonometry the practical problem side should also be emphasized. I would greatly abbreviate reduction of trigonometric puzzles to simpler forms and would only require memorizing of a very few fundamental formulæ. I would wholly omit solution of oblique triangles. As given in most of the text-books this is purely a mathematical recreation. When an actual oblique triangle is to be solved as in civil engineering, navigation or astronomy by the actual workers in these fields the usual text-book methods are not used and certain set forms and tables are used instead.

In the case of geometry I believe in covering a comparatively small amount of ground thoroughly. In this as in all cases effort must be made to give a thorough training in fundamental principles, remembering that this will leave a permanent impression on the mind when detailed demonstrations which have been learned have been long forgotten.

Of solid geometry I would give only that portion relating to mensuration and wholly omit all of the remainder.

Spherical trigonometry is occasionally made a high school subject. This I do not believe should be done as high school pupils are not of sufficient maturity to grasp it properly. Furthermore, it has little or no use in industrial life.

As has been obvious from the paper, the most important

point to my mind in this matter of the relation of secondary school mathematics to industrial life is the omission of detailed mathematical instruction for non-mathematicians and concentration of it for mathematicians. I hope some reform will occur in this respect.

WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE CURRICULUM OF THE SECONDARY SCHOOLS? FROM THE POINT OF VIEW OF THE INDUSTRIAL ESTABLISHMENTS.

By F. W. Thomas,

Supervisor of Apprentices, The Santa Fe System.

The most important of all branches of learning is the ability to read. The translation of symbols, figures, letters, words, understandingly and meaningly to the brain by the eye, stands first in importance of "the three R's." The ability to express thoughts in characters or symbols for the edification of others, the art of inscribing such thought or words on paper or parchment—the ability to write, stands second of "the three R's." And then we have arithmetic, the third of "the three R's." Arithmetic, the lowest branch, the primer of mathematics, is the most useful and most applicable of all our branches of mathematics.

From the point of view of an employer of a large number of boys who have attended the public schools of our country, I wish to address my remarks on the assigned subject.

The Santa Fe Railroad a few years ago was suffering from a lack of first-class mechanics. Wages were good, surroundings pleasant, working conditions excellent, and a spirit of uplift everywhere. But there seemed to be no first-class mechanics in the country desiring work. All seemed busy. The management said: "If we can't hire men, we will make them." So with instructions from our vice-president a committee visited all the prominent railroads and manufacturing enterprises in the country, ascertaining what they were doing towards making men. The committee made its report and recommendations and orders were immediately issued to establish an advanced system of educating apprentices, beginning the following month. This duty was assigned the speaker. With no knowledge of the subject and with only our theoretical and practical education, we dived into the work and today we think we have a better sys-

tem of educating apprentices for the trades than any other institution in existence.

We selected first-class, practical mechanics for shop instructors, men who are not only artists in their particular trades, but who are patient, boy-loving leaders, possessed with the faculty and spirit of teaching boys. Each of these men was given twenty-five apprentices in the shop, his sole duty being to teach these boys their chosen trade. The boys were not left to flounder along alone, picking up what they could here and there, thrown entirely upon the mercy of the shop foremen, or the other mechanics, but these instructors were there to start them out the right way, to guide and direct them in every move, teaching them every part of an engine or car or shop tool, and the function of each part, and so on from one department to another until the expiration of the four-year apprenticeship course. Then they were graduated as first-class mechanics, capable of performing any duty or doing any job that should be expected of one versed in their particular trade.

We found, however, that boys coming to us from our public schools were deficient in practical education, making it necessary in addition for our shop instructors to establish what we called our apprentice school, where these apprentice boys were required to assemble a given number of hours each week to study such subjects as we deemed advisable to teach them. In this school room we had to experiment considerably in order to ascertain the exact things that should be taught and how they should be taught. In the first place, we found there were no text-books adaptable to the work and it was incumbent upon us to prepare such books. We adopted a looseleaf system of lesson sheets and thus, having a printer and a press of our own, we have been able to revise and add to these lessons as often as occasion demanded. In the second place, we found the boys were not all endowed with equal ability to learn and it was necessary that instruction be individual rather than in a class. To this individual instruction we attribute our greatest success. We do not attempt to teach them all the subjects or branches taught in the ordinary school but we make it a point to teach them such branches and only such branches as they actually need in their chosen trade, and these robbed of all frills and trimmings.

This school room instruction is given during daylight hours and the boy is paid while attending school the same as if he were actually working in the shop.

The newspapers, without any effort on our part to advertise the scheme, gave it a great deal of publicity. In a very short while, the boys began to come to us faster than we could take them in and at this date we have over a year's supply on our waiting list. Our practical instruction of the boys in the shop has been quite easy, although we experienced some difficulty in getting the right kind of men to act as shop instructors. It only required patience, care, and individual instruction to make each of these boys competent for his particular work.

In our school room work, about the first thing we noted was the deficiency of the boys in mathematics. The boys who had passed through grammar school and who had had one or more terms at high school, were practically unfamiliar with the solution of problems in ordinary common fractions, and at times they seemed hopeless. The majority of them could not add or multiply with any degree of accuracy and in the multiplication and division of decimals they were distressingly ignorant. The few who were familiar with the four fundamental operations of arithmetic were lost when it came to independent thinking. With a little help, some of them did fairly well, but through it all they showed that in their school work their difficult problems had been solved, or at least the solutions had been started by others.

I have realized more than ever before the importance of thoroughness in the elementary schools. In the various trades mathematics is the most useful of our educational branches and arithmetic is the most important, most practical of all mathematics. Of one million computations, twelve will be found to involve higher mathematics such as calculus, geometry, trigonometry, or algebra, but the other 999,988 require arithmetic only, and I believe that 90 per cent. of the men in life today can get along safely in all their mathematical requirements with a well-grounded knowledge of arithmetic. The present method of teaching writing has made a generation of poor penmen. I sometimes wonder if this is not one reason for such poor adders and multipliers and dividers. The careless method of making

figures, some of which the boy himself cannot read a moment after writing, should make us use our most powerful and effective influence in persuading our public schools to teach the pupils to make good, clear, distinct figures. I have noticed in our apprentice system that the boy who is neat and clean in the school room, who keeps his drawing board and paper clean, who makes clear and distinct lines and letters, is a clean, exact, and neat workman in the shop.

The principal of one of your schools in this city some time ago compiled some very interesting figures from the reports issued by the government on education, and I noted that out of one hundred boys entering high school, only seven enter college and of these seven only one is graduated. I do not mention this with any feeling of animosity against the college but I quote him for the simple reason that the ninety-three who cannot go to college demand of you and of me an effort in their behalf. For one hundred years or more the aim of our public schools has been to prepare the boy for college. Now, what about those ninety-three boys who do not go to college? The progressive feeling is alive in many fields other than politics.

A boy came to us one day with a brilliant recommendation from his principal. He had led his class from the primary grade on up through the grammar school and had had one-half term at high school. Examining him as an applicant for apprenticeship, I found his writing very illegible. While his English was good and he was familiar with the men and places of the city, and could converse intelligently on current topics, his ignorance or carelessness in simple arithmetic was truly distressing. Simple problems in addition and subtraction of fractions seemed a heavy load. He spent as much time adding $\frac{1}{4}$, $\frac{1}{2}$, 7/64, $\frac{3}{8}$, and 9/16 as you would in eating your dinner, and then he didn't add it up correctly. I will say, however, for this boy that he could tell you very quickly how much $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and $15\frac{1}{2}$ dollars amounted to; but put the same fractions in inches or ounces or pints, and he was simply lost. This boy had not been taught the real value or science of numbers. I don't care what process, what method, what system you employ, so long as the child is intelligently and understandingly taught. But you know, and I know, that under the present system the fruit is faulty and warrants a change.

I do not fully agree with the German schools in giving all kinds of mathematics to a boy before he finishes high school, for I do not think that the boy's brain is sufficiently aged to reason with the higher mathematics such as analytical geometry and calculus. I am, however, strongly of the opinion that the teachers should enter more closely into the boy's life and should make a strong effort to find out the boy's particular talent and that the course should be sufficiently flexible to allow the teacher to direct the boy along the channels which such investigation demands or requires.

There is no doubt too much class instruction and not enough individual instruction. Because a boy may be deficient or lagging in arithmetic does not necessarily mean that he is a dunce. He may be very proficient in art; he may love history, he may love carpentry. It is the duty and function of the teacher to find out such facts and then proceed along the lines justified by such an investigation. I am a strong believer in the departmental method of teaching mathematics. Let one teacher teach all mathematics above the third grade in the elementary school, just as we now have teachers for mathematics in the high school, and these teachers should be men. While women are very proficient in teaching other studies, my experience, and I have had teachers of all kinds, is that a man can more successfully and more understandingly teach mathematics than a woman. This is one reason, I think, why mathematics is much more interesting after you enter college than when you were attending public school. I believe that the pupil should begin arithmetic in the third grade and continue on through high school, beginning in algebra the last term of the first year in high school, and continuing for a year and a half, then taking up plane geometry for a year, and finally winding up with on term of commercial arithmetic. I do not think the high school should attempt to teach analytical geometry, trigonometry, or even solid geometry. With the exception of one or two of the higher professions the high school mathematics will equip 90 per cent. of our citizens with all the mathematics they need in life. It is all the farmer will ever need; it is all the contractor will ever need; it is all the banker ever uses. Addition seems to be all the lawyer or plumber uses.

My twelve-year old boy a short while ago brought his arith-

metic home one evening for the first time. He had some problems in board measure and these problems had floored him. He wanted me to explain board measure to him. I spent one hour explaining to him what was meant by board measure and by a board foot. I went to our work bench and explained by different sized boards what was meant by this standard of measurement. I also showed him some bills for lumber and how they figured it out and what was meant by $21.65 per thousand. The text book said a board foot was a piece 1 foot long, 12 inches wide and 1 inch thick. Neither the text book nor the teacher made it plain why or wherefore this standard, nor was it made plain how to proceed if the board was 9 inches wide and 3 inches thick. Now that boy is not a dunce nor a laggard. He is far above the average, receiving perfect marks on ten of his twelve studies, but on this subject he was simply plugging away, blindly trying to get the answer. Now, the schools of Kansas are equal to those in Philadelphia or Boston, if anything, better. They have not been running so very long and their history is short. They are not slow about adopting precedents and have few land marks they hold dear and no sacred customs to keep alive. I am perfectly willing from my experience in dealing with the boys from the grammar schools, to wager my month's salary against the price of a dinner that 75 per cent. of the boys from the eighth grade schools in Philadelphia cannot figure in three minutes how many eggs to give for 20 cents if they are selling for 23 cents a dozen.

I notice in all our arithmetics that the problems generally have an answer in even numbers, while in daily practice there are few problems that come out evenly, few whose solution does not require a ready and thorough knowledge of fractions, and sufficient judgment to know when to drop the fraction, when to approximate its value, and when to carry it in full. Why our text books ignore this, I cannot understand, unless it is a matter of convenience and ease for the teacher and the pupil or to enable the class to skim over more ground.

I like the German idea of teaching, in that the teachers are given some leeway and are not bound alone to the text book but have the privilege of closing the text book for an hour or for a day. The teacher is one who is sufficiently resourceful to add one or a dozen pages and omit the same if circumstances

or discretion calls for it. I believe it would be better if all the mathematics was taught in school and the child not allowed to carry his text book home. There are so few heads of families who can successfully teach mathematics, who can explain the problem or the process sufficiently to make it clear to the boy. Working problems at home, or, as is generally the case, having someone else work them and then handing them in to the teacher may cause the class to move along faster, but it also makes some very stupid mathematicians.

I do not care so much about your text books but I do care about your teachers. The teacher should be sufficiently intelligent and resourceful to invent new problems, to make them so attractive as to interest the child and to make them so fresh and homelike as to keep the child's mind fixed. In nearly every section of the country arithmetic ends with the eighth grade. Now, the average child is just reaching the age in which the beauty of the science of mathematics is dawning upon him, an age at which the use of mathematics is becoming more apparent. In fact, few boys under this age really understand anything about the principle or the science of arithmetic. That is why I urge that arithmetic be continued on, through the high school.

To the great majority of our citizens the struggle for life begins at the age of fifteen. The door of the elementary school has closed for them, the door of the high school opens only to the favored few. The competition for daily bread drives the half grown boys and girls into the market. They take what they find. True, the question of the child's future has peered out of the background in the minds of parents and relatives. Their eyes are fixed on the necessities of the moment. Positions are valued at the salary they offer, however unfavorable the conditions may be for intellectual or moral development. Some few have the force of character to struggle through untoward circumstances. Their intelligence, their will-power, perhaps also their home training, gives them strength to overcome the forces that drag men down. Some few have the good fortune to get into a factory or shop that has a natural interest in well-trained workmen. Some few find employers who do not regard the young hand as a cheap workman but as a human being who must be educated. But the innumerable mass of weaker and less fortunate youths, of whom thousands and thousands are

also valuable human material, and the innumerable mass of real capacity that find no warm-hearted employer and no employment demanding intellect, drift like shipwrecked men on the stormy ocean. Some reach the haven, after a loss of many years; the majority lead a life never brightened by the sun of joy in work. No one has ever taught them to seek the true blessing of work. No one has ever taken the trouble to point them to anything farther ahead than the daily task by which they must earn their bread their whole lives long.

Our present schools are largely like the old school. Years ago we had a rural population and few boys were destined for the professions. We had boys made resourceful and industrious by farm work, boys who were homogeneous sons of American parents, boys zealous for learning in preparation for life's occupations, boys destined for a simply social order, with few occupations and few problems. Today we have cities with a population of all sorts of boys without any occupation, city and town boys who have never performed a single task, children of every nation under heaven, thousands of boys unambitious and purposeless, and a highly complex social order, with innumerable activities and interdependent problems. It is proper for our schools to take these boys and make them the best citizens possible. The revolution of our social and industrial condition makes it more necessary than at any former period of our existence. It is a fair proposition that the school should study its raw material and the kind of a product the market needs and it should endeavor to deliver 100 per cent. marketable goods.

Any boy or girl who can read intelligently, who can write a clean, distinct hand, and who is capable of quickly and correctly performing the four principal functions of arithmetic together with percentage and ratio and proportion, can always get a position. Business men complain continually that the average boy and girl coming to them from our public schools is wholly unable to perform the above functions correctly. Now, since the great majority of such boys and girls must leave school and go to work at the age of fifteen, is it not our duty to so drill, so teach, and so instruct them that they may in a measure be enabled to go out and make a living? It is our duty. It is an obligation for which we are paid and which we owe to the parent and to the commonwealth.

THE CURRICULUM OF MATHEMATICS IN SECONDARY SCHOOLS.

By G. W. Evans.

The high-school subjects in mathematics are algebra and plane geometry. Changes in content or in method, if any, should be made for purposes that may be enumerated as follows: first, to secure more perfect continuity with the elementary school work; second, to prepare more perfectly for comprehension of mathematical science in general; third, to secure the pupil's spontaneous interest by giving him as wide command as possible of such utilitarian problems as are likely to enlist his ambition.

No fundamental change should be attempted, at least until the enterprising and well-informed teachers of the country are united in advocating it. Only two or three changes are suggested here, and if they seem fundamental, they should be taken rather as hopeful prophecy than as militant evangel.

For the first purpose, to promote continuity with elementary school work, more emphasis could be placed on numerical manipulation. Grant that computation is not mathematics, that a wonderfully facile computer may be a mathematical idiot; it is nevertheless true that the pupil feels less like a lost soul if he can hitch the new star to his old wagon. There is a feeling of security if these general quantitative truths can be actually reduced to numerical statement for particular cases.

This work in computation will likewise increase the pupil's practical command. Nothing in American public school education is quite so narrow as elementary arithmetic, confined, as it mostly is, to counting-room details. Decimals that are not dimes or coppers are very unusual. Let us say that a pupil should carry away from his high school work the ability to compute the capacity of his own coal-bin. If you think he can do it on entering the high school, try it. Give him dimensions to the nearest half-inch, and the weight of coal per cubic foot to hundredths of a pound. He is as likely to fill it with a shipload as with a cartload. It is our business to teach him in the first place to use his common sense as a check on his figures, and

in the second place to limit the work of computation and the pretended accuracy of the result by the degree of accuracy of the data.

A further step for the same purpose could be such a change in the order of topics as making the rectangle-area one of the beginnings in geometry. This is one of the very few facts in geometry which the pupil brings with him from the elementary school. He knows it. As a general thing, he knows no reason for it, and yet it furnishes the most available illustration of fractional multiplication, and has a cogent proof that can be unfolded as he progresses. He is mystified rather than helped by the usual pedantic approach to this remnant of his childhood's knowledge.

Moreover this theorem can be used as a foundation for the similarity theorems, as well as for the whole subject of mensuration; so used, it will help to unify a subject which, Euclid's worshippers notwithstanding, does seem, to a beginner, heterogeneous.

As an example of the sort of change that may prepare for the better comprehension of mathematical science in the later work of the pupil, consider the demand made by many for the introduction of the idea of function into high school mathematics. So general a concept will be baffling, if stated as college students have it. But it can be approached through the study of formulæ. Not only formulæ such as mechanics and engineering use, but the treatment of every equation (with more than one letter) as a formula, explicit or implicit. Thus in eliminating by substitution, the pupil obtains an explicit formula for either letter from one equation and substitutes the formula for the letter in the other. This additional technicality has been found in practice to simplify teaching. With the moderate use of graphical methods, and with the introduction of natural trigonometric functions for computing geometrical relations in the triangle and the circle, the progress from formula to function will be gradual and not unnatural. Moreover the practice in algebraic manipulation may be made to appear as a study of different types of formulæ or of algebraic functions. Here again is an advantage in unity; elementary algebra will then appear first as a study of equations, considered as instruments for exact infer-

ence, and then as a study of different types of formulæ, or algebraic functions, some of which have been used for the treatment of equations, and others are studied as new types for their own sake, or for further use in which the pupil is expected to have faith.

The purpose which I have named last, that of securing keener interest on the part of the pupil, seems to me more important than the other two. Sneers at easy ways, and at attempts to amuse, cannot becloud the memories of your own youth; if you worked hard it was not because you were hammered into it, but because you found interest in subjects that repelled others not less able than yourselves. Many that failed could have succeeded; no searcher of statistics can find out what the world has lost by their rejection of mathematics.

One suggestion in the direction of this third purpose is to broaden the treatment of mensuration. Polygons and circles are the only plane areas a high-school graduate can measure; as for solids, he would have to ask a man with lime in his whiskers to estimate a retaining wall for him. Two ideas, mostly new to high school work, may serve to help this need. One is Cavalieri's theorem, embodied in the N. E. A. geometry syllabus; the other is Simpson's rule, familiar for generations to practical men and contemptuously ignored by school-teachers.

Cavalieri's theorem has a proof quite as easy to comprehend as that for the evaluation of π; and it can be so introduced into solid mensuration as to be a foundation for a good deal of the complicated formula-work, and to unify in a very great degree a subject that much needs it.

Simpson's rule is, to be sure, an approximation. It is a good approximation, however, and this fact has a good proof; and the proof of the value of a formula of approximation should not be without interest—even mathematical interest. Moreover it is applicable to solids; the formulæ for the volumes of pyramids and frusta—of the sphere also—being special cases of it. When we use it for solids it is the prismatoid formula, no longer approximate for the solids familiar to high school geometry.

Another, perhaps the greatest, advantage of this rule is the way in which it increases the pupil's command over plane areas. Amateur boat designers can use it to compute their body plans

and deck and water line areas, to locate their center of buoyancy and their center of lateral resistance, to attack the question of stability, and so on. In one book on ship design that I have seen some of the formulæ for these purposes are exhibited with the notation of definite integrals, and are evaluated by Simpson's rule. It may be the beginning of John Perry's millenium, when every boy of fourteen will have some practical facility in the calculus.

High School,
Charlestown, Mass.

WHAT MATHEMATICAL SUBJECTS SHOULD BE INTRODUCED IN THE CURRICULUM OF THE SECONDARY SCHOOL?*

By William Betz.

Introduction.

The discussion of the question before us may be undertaken in two different ways. One may regard the high school curriculum as relatively fixed by traditions and external regulations over which the individual teacher or group of teachers has little or no control. In that case we should merely have to consider the merits of the various existing syllabi. Thus, in algebra we might examine the syllabus prepared by a committee of this association (published in *School Science and Mathematics,* December, 1909). In geometry we now have the National Geometry Syllabus. Other material of this sort is represented by the syllabi of such examining bodies as the College Entrance Examination Board and the Regents of the State of New York. Many good suggestions may also be found in the reports prepared by the numerous subcommittees of the International Commission.†

It seems to me, however, that in spite of the excellence of much of this available material, the very formulation of the question before us implies a different treatment. As a matter of fact, any student of secondary education knows that its most prominent characteristic just at present is a *fluctuation* of aims and values. He also knows that any attempt to solve the prob-

* This paper was read at the Philadelphia meeting of the Association, November, 1912. Since its original presentation, a number of changes were made in its form, in order to adapt it to a wider circle of readers. For the same reason a brief list of references was added.

† Copies of the National Geometry Syllabus and of the reports of the International Commission on the Teaching of Mathematics (American branch) may be obtained gratis by applying to the Commissioner of Education, Department of the Interior, Washington, D. C. Attention may also be called to the Syllabus of Mathematics, prepared under the auspices of the Society for the Promotion of Engineering Education. Ithaca, N. Y., 1912.

lems of one department of instruction without keeping in mind the educational situation as a whole is bound to fail. From this point of view all existing syllabi, even the best, are merely provisional.

Moreover, every intelligent teacher will demand satisfactory criteria by which he may judge for himself the value of proposed plans of reorganization. Such criteria can be derived only after a careful study of the whole educational field.

If this is true, my answer to the above question would be meaningless without a brief analysis, however imperfect, of those modern educational tendencies and views which seem to have a controlling influence on the curricula of our secondary schools. In the light of this analysis I shall then try to state the principal aims of mathematical instruction. On the basis of this double inquiry it will be possible to derive a few guiding principles for the organization of the mathematical curriculum.

The Drift in Secondary Education.

The one dominant impression which a study of secondary education conveys is that of a rapidly shifting background. In a recent number of *Science* (November 1, 1912) Dr. W. J. Fisher, of Ithaca, N. Y., published the result of a statistical investigation of the "drift in secondary education." His figures are based on the reports of the U. S. Bureau of Education. The period covered extends from 1890 to 1910. Dr. Fisher arrives at conclusions such as the following:

1. "While the population of the continental United States has increased 50 per cent., the proportion of the population in the secondary schools has been multiplied by about *three*.

2. "While the proportion of students completing the secondary course and graduating has slightly but decidedly increased, the proportion of them preparing for college, either classical or scientific courses, has been diminished by about 60 per cent." (In 1890, 1900, and 1910, the percentages of secondary students preparing for college were 18.66, 14.53, and 6.8 respectively.)

3. "The tabular and graphical representations of statistical facts show at a glance that since 1890 the problem of the secondary school has changed from that of the fitting school to one of a decidedly non-fitting school—some bigots would say a decid-

edly *unfitting* school; a school in which only 6.8 per cent. of the pupils anticipate college work of any sort. This being the case, the colleges and universities can not lead the way in the fashion of 1892 and the Committee of Ten; the problems of secondary education can be solved only in the schools."*

The first great fact to remember, then, is that a beginning has been made in the emancipation of the high schools from college domination. What stand shall an open-minded teacher take in this matter? Certainly the demand for high school autonomy cannot be lightly set aside in the face of the above figures.

The criterion of readjustment for a given subject would seem to be that just as soon as a considerable number of efficient teachers reach the conclusion that the college requirements in that subject are out of harmony with the best interests of the majority of the pupils pursuing that subject, the high school should travel its own road irrespective of the college. Such declarations of independence have already been issued by teachers of English, of foreign languages, history, science, and other high school subjects.

Unfortunately, neither of the parties to this quarrel has avoided misrepresentation. We should not forget that we owe to the colleges a great debt of gratitude for furnishing to our secondary schools the first reliable standards of attainment at a time when the absence of such standards would have meant chaos. I believe also that the colleges are very desirous of helping the secondary schools, and that we shall continue to look to the colleges for intellectual guidance. It should not be forgotten that, in general, the American standard in secondary education is still considerably below the European standard. It is an open secret that our present policy of extending every educational opportunity to the masses has brought about a further lowering of standards. This may be unavoidable, but it does not make the fact less serious.

Obviously, the colleges cannot do their work without ade-

* These inferences, like all conclusions based on statistical evidence, must be taken with a grain of salt. Some schools even now are almost exclusively college preparatory, and perhaps justly so, in view of the large number of students entering college from these schools. Each community must study its own educational needs on the basis of local statistics.

quately prepared students. Besides, it is by no means certain that the best educational policy of a country consists in ignoring or neglecting the training of even the few who are destined to be the intellectual leaders. At the same time, the colleges have sinned grievously through their ignorance of modern high school conditions, through emphasizing quantity and technicalities rather than insight and quality, and through failing to supply the secondary schools with teachers having the necessary breadth of training and the vision to cope with the new situation. Such furious attacks as that made by Mr. Munroe may impress one as a little ungenerous, but they will probably go a long way toward accomplishing the necessary reforms.*

"New Demands in Education."

Most of the "new demands in education" may be grouped around three distinct ideals. These are the ideals of individualism, of vocational training, and of social efficiency. In reality they are but three aspects of the one underlying tendency away from uniformity by emphasizing personality.

Individualism.

Individualism is the characteristic feature of modern times. The student of history could easily trace its development, first in the intellectual field, as shown by the renaissance movement, then in the religious field, during the great Reformation movement of the sixteenth century, then in the political field, as indicated by the great political upheavals of the last three centuries, and finally in the industrial field, as is evidenced by the present struggle for economic freedom. In its extreme form this tendency leads to egoism and to the questioning of all restraints of personal freedom. One writer gives it the beautiful Greek name "eleutheromania," which he defines as "the instinct to throw off not simply outer and artificial limitations, but all limitations whatsoever." He goes on to say that for a century the world has been fed on a steady diet of revolt.†

The school, we are told by the individualist, must treat each child according to his own native capacities. All his latent pos-

* See J. Ph. Munroe, "New Demands in Education," Doubleday, Page and Co., 1912. Chapter 13: "How the Colleges Ruin the High Schools."
† Irving Babbitt, "The New Laocoon," 1910.

sibilities of body, mind and soul must be conserved and developed, and not dwarfed.

Vocational Training.

This individualized pupil, so we are informed, must be prepared for life.

But for what sort of life? Since college preparation is to be largely ruled out, the answer commonly given is: for social, commercial, and industrial life. Preparation for a vocation is the watchword of many an ardent reformer. But *which* vocation is to be recommended to the individual pupil?

The difficulties we encounter at this point are enormous. Thus far, a satisfactory scheme for industrial education is yet to be announced. What may be done and perhaps should be done can be inferred from such sources as Leavitt's Examples of Industrial Education, Munroe's New Demands in Education, and the bulletins of the National Society for the Promotion of Industrial Education.*

Efficiency.

The problem of vocational training will eventually be solved, but perhaps not in our day, if only for financial reasons. Meanwhile, we are assured that the new high school will be subjected to a rigid test of *efficiency*.

This demand should be welcomed enthusiastically by all teachers. However, in many of the current efficiency arguments there is an element of charlatanism, of gross materialism, and of loose thinking. Some efficiency experts would have us think that the work of a school can be gauged like the output of a factory. It should not be forgotten that the finest and most lasting results of good teaching cannot be tested by the methods of the counting-room or measured in dollars and cents, and in

* See also the bulletins of the National Association of Manufacturers of the United States of America (170 Broadway, New York); the very extensive report of the N. E. A. committees, printed in the 1910 volume of the Proceedings of the National Education Association; the bulletins of the U. S. Bureau of Education; Report on Vocational Training in Chicago, City Club of Chicago, 1912; G. Kerschensteiner, "Education for Citizenship," Rand, McNally & Co., 1911 (prize essay); R. M. Weeks, "The People's School," Houghton Mifflin Co., 1912; Bloomfield, "Vocational Guidance of Youth," and Snedden, "The Problem of Vocational Education," Houghton Mifflin Co., 1911 and 1910 respectively.

fractions of a second. It is the very effort to examine, examine, examine, that has brought the school into such a formalistic rut. Let the examination enthusiasts read what Dr. J. J. Finlay of Manchester University has to say of our essentially British examination system.*

It cannot be denied that there is a semi-humorous inconsistency in the attitude of some of our most ardent school reformers. On the one hand, the school course is to be adapted to the individual. The child is to study only the things that appeal to him. If the English classics are uninteresting, let him read magazines. If algebra or Latin are too hard, let them be dropped. On the other hand, efficiency means the living up to definite standards, which is the very opposite of this perpetual coddling and fondling. A common-sense middle ground is necessary. As Munroe puts it: "Every high school boy is a problem by himself; and the business of the high school is to develop him, as an individual, to his highest possible usefulness as a man and as a citizen. To be so developed, he must, in the first place, be disciplined by hard, serious, steady work; to do that hard work he must be interested in it; and, to be interested in it, he must himself see that it is going to be of use" (p. 210, *l. c.*).

Attempts in Reorganization.

By what means may the high school hope to meet these demands? The answer is threefold.

First, by a thorough study of current criticisms of its present system, followed by a restatement of educational aims and values in the light of past and present educational history. In this way alone can we expect to gain the necessary insight to meet the problem of individual development.†

* "The School," Home University Library, Holt & Co., 1912, pp. 124–131.

† In Johnston's High School Education, for example, 21 serious criticisms against our present system are enumerated. Only one of these can be mentioned here, namely the statement that "enormously large numbers withdraw from school."

What are the facts?

According to a recent report of the U. S. Bureau of Education the number of students enrolled in *public* high schools in 1911 was 984,677. Of these, 421,335 were in the first year of the high school.

Second, by extending the opportunity to acquire some form of secondary education to "all the children of all the people." This will meet the problems of vocational training.

Third, by adopting the six-year curriculum. This is our principal remedy in the direction of greater efficiency.

A discussion of these essential principles of reorganization is impossible at this point. I can only refer to such books as

Sachs, "The American Secondary School," The MacMillan Co., 1912;

Munroe, "New Demands in Education" (see above);

Johnston, "High School Education," Scribners, 1912;

Bagley, "Educational Values," The MacMillan Co., 1911;

Hanus, "A Modern School," The MacMillan Co., 1909;

Hanus, "Educational Aims and Educational Values," The MacMillan Co., 1902.

Concerning a restatement of aims and values, it may safely be asserted that no teacher can expect to do justice to his subject without studying carefully the modern point of view in regard to such questions as the theory of mental discipline. Rüdiger's summary of recent educational theory is very helpful.* Contrary to the usual impression conveyed by superficial alarmists it may be said confidently that "we have neither proved nor disproved the formal discipline dogma" (Johnston, p. 37). It is very probable that the painstaking investigations of psychologists will strengthen the position of mathematics in the curriculum, always with the proviso that a teacher knows what he is really doing. I even believe that we are on the threshold of a return to drill methods.†

"The age census for 1910 is not yet published, but estimates based upon the percentages for 1900 indicate that the number of children 5 to 18 was approximately 25,000,000 in 1910. About 1,824,000 of these, or 7.28 per cent., were in the 14 year group. The 13, 15, and 16 year groups are very close to this number. We should expect this 1,824,000 group to furnish the students for the first year of the high school. We find 421,335, or 23.1, in the first year of the high school in 1911." (Quoted from a letter of Mr. A. Summers, statistician of the Bureau of Education.) These figures tell their own story.

* W. C. Rüdiger, "The Principles of Education," Houghton Mifflin Co., 1910, Chap. VI.

† How does this passage impress the timid teacher who had gained the idea that all drill was forever tabooed:

As to the extension of high school facilities to the masses, thus making the high school a real "people's college," it may be said that at present a conflict is going on between the cosmopolitan type of high school and the specialized high school. Each has its merits and its defects. The cosmopolitan type is better as a selective agency, since it makes a wider appeal to individual aptitudes, while the specialized high school will undoubtedly do better work in its own field.

Finally, the adoption of a six-year curriculum is only a question of time, whichever of the four possible forms it takes. Recent popular and educational discussion is enthusiastically in favor of this reform. Personal inquiry of a score of partial or complete adoptions of the plan in the United States brings me the conviction that it is meeting with all the success which could be expected under the circumstances.*

Aims of Mathematical Teaching.

Having now completed a hasty examination of the educational background, we may proceed with more assurance to a consideration of our main question, provided we can agree on the aims of mathematical teaching in our secondary schools.

"The cry for high school freedom from domination, even when justified, has carried with it the tone of impatience in directing mental processes all of which must be slowly, painstakingly, and expertly supervised if they are to be permanently worth while. Insolent and careless intellectual attitudes, superficiality and lack of thoroughness or of finished knowledge of a few fundamental subjects, inability and disinclination to think an issue or a problem through to its minute details, halfway racings into utilitarian fields, a tendency to tackle anything but the trunk of the curricula, novelty specifics in which pedagogic mastery of the new material is not assured, all characterize questionable traits in high school graduates and suggest that we must regain our faith in the ultimate values and permanent results of drilled training such as some of the older disciplines and traditional models seem still uniquely to afford." (Ch. H. Johnston, "High School Education," Chap. 2, p. 31.)

* See J. Sachs, "The American Secondary School," p. 108, The Macmillan Co., 1912; Ch. H. Johnston, "High School Education," Scribners, 1912, Chap. IV.; Bulletin, 1911, No. 16, whole number 463, United States Bureau of Education, containing a report on the six-year curriculum from the standpoint of mathematics; *The School Review*, December, 1912 (pp. 665-688), January, 1913 (pp. 1-25); *The Ladies' Home Journal*, February, 1913, March, 1913, April, 1913.

Mr. T. Brookman, in his splendid article on first-year high school mathematics,* says with only too much truth: "It is difficult to find an instructor in mathematics who has thought out the problems of first-year algebra and why it *is* taught or why it *should* be taught to all students. He only knows that he is thankful that the subject does not fall to his lot." Does not this ignorance account for many of our troubles?

The aims of mathematical teaching center around the two ideas of insight and efficiency, which may also be described by the terms impression and expression, or by form and content. The one is subjective, the other objective. With this main classification in mind, we may distinguish these five important aims of mathematical teaching:

1. *Symbolism.*—The pupil must be made to appreciate the value of mathematical symbolism as man's chief tool for summarizing important quantitative relations. He should respect a *formula* as a condensed record of human effort and achievement. Mathematics furnishes a language without which, it is not too much to say, our present civilization cannot be understood and cannot exist.

2. *Applications.*—A living language like mathematics exists for the expression of ideas. Hence, the pupil should be impressed with the reality of mathematics by numerous applications lying within his grasp.

3. *Function Concept.*—The function concept, together with the equation, should be given a prominent place. Training in functional thinking is almost a necessity for the modern mind. It is at the bottom of the doctrine of evolution. Without it modern science remains a jumble. More than that, the function concept furnishes a unifying bond for all branches of mathematics.

4. *Space Intuition.*—There should be constant insistence on a cultivation of space intuition. The training obtained in this way is not only extremely valuable for mathematical purposes, but is also indispensable to the mechanic, the designer, the representative artist.

5. *Logic.*—The pupil should learn to understand and to appreciate the logic of mathematics, and he should show evidence of

* *Teachers College Record,* March, 1909.

this training in all his other work. As a major aim this is second to no other, a fact which has been recognized since the days of Plato. The constant necessity of checking results and of giving reasons for statements is conducive to accuracy in an incomparable way. Besides, the habitual use of inductive and deductive reasoning in mathematical teaching familiarizes the student with the two principal modes of discovering and of testing intellectual truths.

It is very doubtful whether any other high school subject can produce equally cogent and permanent reasons for being included in the curriculum. This becomes all the more apparent if it is observed that each of these five aims is essential from either a purely ideal or a narrowly utilitarian standpoint. To borrow a happy phrase of Professor D. E. Smith, mathematics is to a remarkable degree "potentially practical."* It may be said without hesitation that if the opponents of mathematics should ever succeed in abolishing it from the curriculum, public opinion would demand its restoration within a year. Mathematics meets a fundamental human need.† Finally, let it be remembered that ours is an age of mental unrest and uncertainty. To acquaint the minds of the young at this critical period with reliable standards of intellectual certainty is a service no less real because it is often overlooked, a service that endows mathematics with the highest moral value.

The Mathematical Curriculum.

Unfortunately there is very little doubt that the mathematical curriculum at present is not doing justice to the five aims enu-

* *Bulletin of the American Mathematical Society,* February, 1913, p. 250.

† For a discussion of the old conflict between realism and idealism in mathematical teaching see D. E. Smith, "The Teaching of Geometry," Ginn & Co., 1911, pp. 7–25; Th. J. McCormack, "Why Do We Study Mathematics: a Philosophical and Historical Retrospect," Cedar Rapids, Iowa, 1910 (an abstract of this paper may also be found in the N. E. A. Proceedings of 1910); W. H. Metzler, "Educational Value of Mathematics" Journal of Pedagogy, June 1905, "Formal Discipline," This Magazine, September 1910, "Mathematics for Training and Culture," Proc. Ass. Coll. Prep. Schools Middle States and Maryland 1910. See also the preceding number of *The Mathematics Teacher* (March, 1913).

merated above. The chief hindrances preventing their more perfect realization are the compartment system of teaching, the four-year curriculum, and the poor preparation of pupils and teachers.*

I shall begin this brief discussion of the mathematical curriculum by quoting Professor Felix Klein's definition of elementary mathematics: "In all domains of mathematics we may call those parts 'elementary' which may be understood by a person of average ability without long continued special study."† He limits this definition, however, by assigning to the school those elementary topics only which are necessary to a comprehension of our modern civilization. In the opinion of Professor Klein this comprehension can best be secured by making the function concept in geometric form the very heart of all mathematical teaching (p. 34, *l. c.*).

It will be seen that this broad definition still leaves a considerable amount of freedom to the individual teacher. This freedom is checked chiefly by the teacher's limited preparation and by the requirements of examining bodies. Besides, each country in the course of time has developed its own mathematical routine which it would be difficult or useless for the individual teacher to ignore. Reforms, accordingly, must be a matter of evolution.

With these limitations in mind, I submit the following topical outline in synoptic form.

(In arranging the algebraic topics the original intention was to show the dependence of every topic on the preceding topics in the same column and in the adjacent columns. This plan was carried out but partially. Otherwise the table is self-explanatory.)

* See Report of International Commission, Bulletin 16, 1911, p. 94; Th. J. McCormack, "Education, Utopian and Real," *School and Home Education,* January, 1913; J. E. Russell, "Professional Factors in the Training of the High School Teacher," *Educational Review,* March, 1913.

† Klein und Schimmack, "Der Mathematische Unterricht an den Höheren Schulen," Teil 1, p. 111.

OUTLINE OF MATHEMATICAL WORK THAT SHOULD BE REQUIRED OF EVERY STUDENT IN A GENERAL HIGH SCHOOL COURSE.

Elementary Algebra.

I NUMBER-SYSTEM.

1. Positive Integers.
2. Negative Integers.
3. Fractions.
4. Irrational Numbers.
5. Imaginaries.

II OPERATIONS.

1. Addition.
2. Subtraction.
3. Multiplication.
4. Division. (H. C. F., L. C. M., Factoring.)
5. Involution.
6. Evolution.
7. Logarithmation.

III EQUATIONS.

1. Simple.
2. Fractional.
3. Simultaneous Linear.
4. Radical.
5. Quadratic.
6. Exponential.

IV FUNCTION-CONCEPT.

1. Statistical Graphs.
2. Graphs of Formulas.
3. Graphic Solution of Problems.
4. Graphs of Linear Functions.
5. Graphic Extraction of Roots.
6. Graphic Solution of Quadratic and Higher Equations.

V APPLICATIONS.

1. Arithmetic.
2. Geometry.
3. Mechanics.
4. Science.
5. Economics.
6. Statistics.
7. Shop Mathematics.
8. Slide Rule.

Geometry (Plane and Solid).

Symmetry, Congruence, Equality, Similarity. (Follow National Geometry Syllabus.)

Trigonometry (Plane).

(Follow suggestions of American Mathematical Society. See Syllabus of College Entrance Examination Board.)

In the preparation of the above table only a desirable minimum requirement was kept in mind. For a discussion of the separate topics I may refer to the report on "Mathematics in the Public and Private Secondary Schools in the United States," prepared by Committee III of the International Commission (Bulletin 16, pp. 15–75.) Hence I can bring these considerations to a close by adding the following guiding principles for the administration of the mathematical curriculum, as well as some statements that may serve to justify the extension of the above table beyond the customary requirements.

1 The mathematical curriculum of the future must meet the wants of all types of students. For some of these types only

actual experiment can determine the most desirable selection and sequence of topics.

2. Each topic should be viewed in the light of Professor Klein's definition of elementary mathematics.

3. The five aims stated above must be met at least approximately, if the cultural and the practical value of mathematics are to receive due weight.

4. These five aims cannot be fully realized unless every student in the general course is required to become familiar with the elements of algebra, of plane and solid geometry, and of plane trigonometry.*

* If any one should consider this an unreasonable demand, let him make a brief study of the European requirements. If America wishes to compete with Europe along industrial, technical, or scientific lines, it cannot ignore the increased educational opportunities given to European secondary students. In this connection the following passage may be of interest (taken from a report of Professor D. E. Smith on the educational work of the Fifth International Congress of Mathematicians, held at Cambridge, England, last August):

"For example, we have generally failed to do anything worth while with algebra and geometry in the elementary school, altho there is hardly a country among those reporting at Cambridge that does not make a success of this work. They do it by cutting out most of the subjects treated of in the last two years of our arithmetic course, and putting the classes in the hands of the same teachers of mathematics that are to present the subject in the years corresponding to our high school. This enables them to prepare the pupil's mind for a serious study of algebra, geometry, and trigonometry in the next two years, and to cover much more ground than we do. Furthermore, they are not content with stopping at quadratics and plane geometry, but it is a growing custom to add to the solid geometry and trigonometry a course in analytic geometry and the calculus in the secondary school, a custom that is pretty sure to be adopted in such of our schools as have teachers capable of understanding it and superintendents sympathetic with progress. If it is said that the calculus must be very superficial if done by pupils before entering college, it may be replied that in one large English school for boys, recently visited by the writer, about half of the calculus written by Professor Osgood was covered, the examinations being set by outside authority and the results being in every way satisfactory. Moreover, the subject was not elective but was required of every boy in the school. It is needless to say that this was exceptional, but good courses, even tho of lower grade than this, are given in many schools, and there is a growing literature upon the subject. Our courses are more systematic than those found in the Teutonic countries, resembling somewhat more

5. The function concept in geometric form should be regarded not as an additional topic, but as an organizing principle of the highest value.

6. *Solid Geometry* is necessary to secure training in space intuition, and to open the door to numerous applications. The teaching of plane geometry alone gives the pupil a false impression. We live in a world of three dimensions, and not in a "Flatland."

7. Trigonometry is necessary for its great practical value in all departments of mechanics and engineering, not to mention surveying. It also gives the pupil valuable numerical training in the use of tables, and it affords opportunity for a thorough study of logarithms and of the slide rule.

8. The topics should be so selected and arranged that the psychological element in the development of the pupil is kept in mind. To a certain extent this order coincides with the historical order. There should be a transition from the concrete to the abstract. Great attention should be given to the time element in the assimilation of new ideas. Just as arithmetic furnishes the necessary background for algebra, so we should have a preliminary course in plane geometry, in solid geometry, and in trigonometry. The experience of a century in European schools proves the correctness of this view.*

9. Any principles or topics which cannot be given adequate scientific treatment, or which have too little significance to the

those found among the Latin peoples. For example, our geometry is more systematically arranged than that of Germany, resembling more that to be found in France or Italy. The blind attack upon all mathematics, which seems to be a phase of degeneracy in some of our educational circles today, is not found in the best European countries. On the contrary there is a general desire on the part of all to hold to the best that we have, while constructively modifying the weaker parts of the edifice." (*Educational Review,* January, 1913, p. 3.)

* See P. Treutlein, "Der Geometrische Anschauungsunterricht," Leipzig, 1911 (an excellent book, containing a splendid bibliography); G. C. Young and W. H. Young, "Der kleine Geometer," Leipzig, 1908 (a German translation of the corresponding English text by the same authors); C. A. Laisant, "Initiation mathématique," Hachette and Co., Paris, 1906; G. Veronese, "Elementi di Geometria intuitiva," Padova, 1906; D. E. Smith, "L'enseignement of Mathématique," November, 1912, pp. 507–527; W. Betz, "Intuition and Logic in Geometry," *The Mathematics Teacher,* September, 1909, p. 3.

pupil, should either be assumed without proof, or be omitted entirely. To this category belongs such material as the "incommensurable case,"* the theory of limits, cube root, complicated work in factoring, in fractions, in involution and evolution.

10. The pupil should gain a conception of the significance and the organic relation of the principal divisions of the subject. Whether this is to be done by a fusion of the different branches of mathematics or by expert correlation, must be determined ultimately by more careful and extensive experimentation.

If one prefers to begin with an unsystematic "problem course" under the general name of "mathematics," care should be taken that a system of some kind is developed in the end. Otherwise the result will invariably be confusion and inefficiency. On the other hand, the usual systematic courses should be so arranged that contact with life is not lost and that the pupil is not given a wrong impression at the very outset by useless technicalities. In general, a minimum mathematical basis with which the pupil has become perfectly familiar is vastly better than a mass of unassimilated material.

11. The course in algebra should represent the organic interweaving of these five elements: the number classes, the operations, the equation, the function concept, and the applications.

If a comparison be permitted, it may be said that the number-system of algebra furnishes the raw material, the *vocabulary*, of our mathematical language. The operations represent the transformations of declension and conjugation, thus creating flexibility of *form*.† The equation is the *sentence* of algebra. It is the chief tool for the statement and solution of problems. But the function is an entire paragraph, the development of an idea. The equation is only an incident in a functional development. The function is dynamic, while the equation is static.

* For a discussion of the "incommensurable case" see the National Geometry Syllabus, p. 40; also Arthur Schultze, "The Teaching of Mathematics in Secondary Schools," The MacMillan Co., 1912, pp. 188–196.

† It would be possible to extend the linguistic analogy still further by introducing at this point another division of algebraic work which may be said to correspond to the syntax of a language. Into this division could be placed the fundamental postulates and the typical processes and rules, such as the rule for squaring $a+b$ and the rules for factoring polynomials.

And just as an essay must have a worthy theme, so this mathematical language can display its power only by effective applications. The problem work of algebra corresponds to the writing of complete essays. The teacher of English introduces different types of composition and treats a great variety of themes. The teacher of algebra will try to secure mathematical fluency by a sufficiently elastic assortment of problems.

12. The topics of geometry group themselves organically around the fundamental ideas of symmetry, of congruence, equality, and similarity. The locus problem should receive very much greater emphasis, for it represents the dynamic, functional element of geometry. Above all, opportunity should be given not merely for a reproduction of book theorems, but also for the independent discovery and proof of geometric truths.

Conclusion.

In short, there is no reason why the curriculum in secondary mathematics, if properly administered, should not create in a large number of pupils some of that genuine enthusiasm and interest with which mathematics—the most perfect of all sciences—has never failed to inspire the leaders in the long upward trend of human development.

East High School,
Rochester, N. Y.

WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE CURRICULUM OF THE SECONDARY SCHOOLS? FROM THE POINT OF VIEW OF THE COLLEGE.

By E. S. Crawley.

Mr. President, Ladies and Gentlemen: Had I followed the orthodox method in making preparation to speak on the topic here assigned me, I should no doubt have devised an elaborate questionnaire and sent it broadcast to my colleagues in this district. By so doing I should have made myself a source of annoyance to a large body of busy men, and have been able to come before you today with a mass of statistical matter showing the percentages of college teachers who favor this or that topic in the preparatory school curriculum. The questionnaire habit has however, become too prevalent. I dislike to receive these documents and have a lurking suspicion that others may have a like feeling, so I have refrained from adopting this method. Moreover, I do not believe that such procedure would have led us to any very satisfactory conclusion. The question immediately before us: "What mathematical subjects should be included in the secondary school curriculum—from the point of view of the College?" that is, with a view to mathematical work in college, is one which it seems to me can be answered categorically in only one way and that very briefly. When we say that what the colleges ask is that the fundamental principles and processes of mathematics be taught so far as they are set forth in the study of elementary algebra and geometry, all has been said that need be said. But you may reply, that is begging the question; what we wish to know is how best to do this. My rejoinder would be that I cannot tell you. The teacher in the secondary school can best decide that for himself. I say this for two reasons, first because what will serve in one case may not be at all suitable for another, and to attempt to prescribe details is for this reason futile. Moreover, and secondly, I consider that it makes very little difference what is taught,

that is, how fully this or that topic is treated, provided always fundamental principles are insisted upon unceasingly and their use and application kept always in the students' view.

It seems to me, therefore, that the way in which I can best contribute my mite to this discussion is by calling attention to certain broad general considerations which in my experience with college students leads me to judge are not sufficiently kept in view in all cases.

Let us consider the matter of elementary algebra. Algebraic principles and processes are really few and simple. It is true that the text books divide the subject into a great many topics but if any of these be taken and analyzed it will readily be found that its treatment goes back to an application of principles already well established. Each topic can be subdivided into various cases and often is so subdivided. There is no objection to this if the underlying principle is always sought out and brought to light. But the teacher who makes a separate thing of each of these different cases in all the topics treated will not succeed, because he will swamp his students in multiplicity of detail. The student will form the habit of relying upon his memory of this or that point of detail for the particular case under discussion instead of simply applying the underlying principle. In this way if he has a good memory he will be able to show a certain superficial success, but he will not have learned anything about mathematics. More than this the possibility of his ever knowing anything about mathematics will be in a great measure destroyed by this process. I think that I have made clear what I mean, but let me illustrate briefly. Take the great question of the simplification of algebraic expressions, which after all embraces a very large part of the work in elementary algebra. Practically all of this is based upon one process, factoring, and one principle, that we may multiply or divide the numerator and denominator of a fraction by the same factor. Again the whole subject of the solution of quadratic equations is nothing more than an application of the fact learned very early in the study of algebra that the square of a binomial consists of three terms one of which is twice the product of the square roots of the other two. In geometry in the same way unless a student is brought to know the principles

of geometric reasoning all his work goes for nought. He may it is true learn a few facts about geometry which will be of more or less importance to him practically, but he will not learn anything about mathematics.

Twice in this discussion the phrase that under specified conditions, a student will not learn anything about mathematics, has been used. In order that we should understand each other clearly it will be well to come to some agreement as to what we are to mean by "knowing something about mathematics." The phrase itself does not necessarily convey the same meaning to all minds. Before stating what in my opinion this phrase should mean for us, let me digress for a moment. This digression as you will see will lead us back to the point at issue. Let us glance for a moment at the subjects which find a place in the curricula of our secondary schools, which apart from mathematics form the bulk of these curricula. We find the greater part of the time devoted to languages, literature and history. Each of these subjects has a certain informational value, but it is not primarily for this that they hold the place assigned them. Is it not because in all of these we find a great human interest, because each in its own way is a record of the way men, the great men of the world, have thought or spoken or acted? I think that therein lies their value. We want to have our boys and girls study about these things because they will learn in this way how men who have gone before and how those now in the world think and act. So it should be in the study of mathematics. This study is important because it reveals how men have thought about the problems which belong to the sphere of mathematics and how they have conquered them. When I speak of "knowing something about mathematics," I mean therefore something of this kind—you can get my meaning from what has been said better perhaps than I can express it—I mean getting some insight into the beauty and simplicity of mathematical processes and mathematical principles, something of the wide extent of the mathematical domain; for we live in a mathematical world far more truly than we ordinarily realize. We have but to consider not only how mathematical laws control the operations of nature, but also how they are at the basis of most of our material activities to see how this is so. And the

keynote of it all is simplicity. The same little "two and two make four" principles run through it all. I call them little not because they are insignificant, but because they are so plain and easily understood. In their application their results are often of tremendous importance.

We hear in these days a great deal about the difficulties of mathematics. Books and papers are written, addresses are made, and associations meet for discussion, all upon this great subject of how difficult mathematics has become for our boys and girls, and about what we should do to make it easier. I believe that all this is wrong because in the first place, so far as any real difficulties are concerned, there are none, and in the second place those things which appear to be difficulties do so mainly from causes which have no essential connection with mathematics. They appear as difficulties owing to lack of concentration, lack of persistence, lack of patience, which is almost the same thing, and probably to lack of imagination. I might add to this, if I were not afraid you would think me too old-fashioned, a lack of reverence for those things and those ways of thinking, simply for their own sake, which have enabled men to reach the place which the world occupies today. The joy of intellectual effort and the satisfaction of achievement in the same field are too little known and valued amongst our school boys and girls today.

Now let us see in a more concrete way in what these so-called difficulties consist. The student faces a complicated algebraic expression, or a problem in geometry the solution of which is not too painfully evident. Does it arouse him to effort, does he see in it something to conquer, something against which to pit his intellectual brawn? Too often not. He has not been trained to do that sort of thing. His brain is flabby from disuse and his ambition atrophied. What is needed first is that he should have enough grit, or "sand" as we call it now, to refuse to be conquered by such a trivial thing, and then that he should go to work patiently and persistently, taking the thing step by step, applying a few simple principles carefully and accurately, and the thing is done. But patience, persistence and accuracy do not appear to be included in the list of school boy or girl virtues. Fortunately there are some who come through

the schools with ability to do this sort of thing. They are perhaps natural leaders in their respective fields of endeavor, but at all events they know what they want to do, and have the energy to make the effort to do it. I fear you may think that I am by implication giving the schools some hard knocks. That certainly is not my object, but I should be falling short of my duty if did not present the matter as I see it. So far as the schools and colleges are concerned I believe that they are manned by a force of men and women who rank considerably above the average in conscientiousness and in devotion to duty, and that they give to their students the best that is in them. Why then should we harp so continually upon this one string of unsatisfactory results? It cannot all be imaginary, nor do I think that the explanation is to be found in any very great advance in our standard of efficiency over that which satisfied former generations working in the same lines, although we have perhaps advanced somewhat in this way. Still the main source of the trouble must be sought elsewhere, and I want to make a suggestion touching this and leave it for your consideration to determine how nearly I may be right.

We are all aware that educational expansion in this country has advanced during the last two or three decades with tremendous leaps. The percentage of the population enrolled in the high schools, colleges, and universities, is continually advancing. Is it therefore not reasonable to conclude that the average of the natural intellectual endowment of students who are now seeking higher education is lower than formerly, and that consequently we must seek the root of our troubles not in our methods but in the material which we have to handle. I believe that this view of the matter is worth thinking about. If there is any truth in it, what are we going to do about it? It is a trite saying that many a good shoemaker is spoiled to make a poor lawyer, and we all know the old proverb about trying to make a silk purse out of certain unpromising material. Are we trying to make too many silk purses under circumstances which forbid hope of success? If so, I repeat, what are we going to do about it? It is easier to ask the question than to answer it, and I am obliged to confess that I have nothing to suggest, and must leave it in your hands. Whatever our conclusion upon this im-

portant and far-reaching question, we must face present conditions as we find them, and to return to the specific question of this discussion, let me conclude by urging again the importance in mathematical work in the schools of the simple inculcation of principles. Teach the students always to base their work on the underlying principles. Banish memory work from mathematics, except for the everyday formulæ, for definitions and for such of the results as require to be fixed in the mind for future reference. Stimulate brain effort. If you can so conduct your work, you will need no prescription as to what to teach in preparation for college beyond the simple one already given,—elementary algebra and geometry.

University of Pennsylvania,
Philadelphia, Pa.

WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE CURRICULUM OF THE SECONDARY SCHOOL? FROM THE POINT OF VIEW OF THE TECHNICAL SCHOOL.

By Arthur S. Hathaway.

After careful consideration of this question, it has appeared to me that any changes in the present curriculum of secondary schools in mathematical subjects should be not in the subjects taught, but in the object and character of the instruction. What the technical student most needs is a working knowledge of arithmetic, algebra, and geometry. He should be able to make ordinary arithmetical computations with accuracy, perform ordinary algebraic operations without making those numerous blunders which result from a confused perception of their arithmetical foundation, and have a store of geometrical facts and principles fixed in his mind by having logically worked many original exercises.

In fact, mathematical analysis is to the technical man merely a tool or instrument for doing his work, of which he must have not only the knowledge and understanding but also the expertness which he has with other tools. The purpose of a variety of topics and problems should be to give the necessary drill in fundamental operations without making the task too dull and grinding, just as in manual training, we give the pupil some piece to make, not for the purpose of having that piece made, but in order that he may learn to use the tools which it requires.

To seek expertness and accuracy in the use of mathematical operations is not opposed to the idea that mathematics is the highest type of logic and reason. No one can become accurate in the manipulation of mathematical symbols without understanding and following some good form of reasoning for every step of the work. The good workman can explain every step of the process by which he does a certain piece of work, how it is begun and how finished, the different tools that should be

used, and why. He can also explain the mechanism of his tools and expound on the principles by which they work. In fact, the technical school requires that knowledge of elementary mathematics which is professional and not amateurish, because it is deeper and more thorough.

There is too much formalism, and not enough of thought and understanding required of the pupil. Students seem to be taught that a proposition is not demonstrated or a problem is not solved, unless it is done in a certain formal way, and any appeal to his common sense is doubtfully regarded. Facts are discovered in mathematics as they are in physics and chemistry. Verification is the best of all proofs.

The student who is thoroughly drilled in elementary arithmetic, algebra and geometry, will find that he has acquired the fundamental basis of all higher mathematical work. Higher algebra and trigonometry are only new pieces which are constructed with these elementary tools. Analytical geometry and the calculus bring the first great change to his method of work by the introduction of problems of variation. The purposes for which he has used the elementary mathematics are changed, and although all of it is still used, he is not clear as to the whys and wherefores. By continually testing the firmness with which the new ideas are built upon thoroughly familiar foundations, he soon gains confidence in the reality of the new outlook, and learns that keen insight into natural phenomena is one of the fundamental developments of the new subjects. For mathematics contains the vital elements of engineering knowledge, so that great mathematicians have often been notable engineers from the time of Archimedes down to the present.

When the question of elementary mathematical instruction for technical schools is answered by the declaration that its leading object should be to produce a pupil who is thoroughly drilled in the elementary processes, and able to use them as a workman does his tools, the other questions as to what topics should be in the curriculum, and whether problems should be of the old or modern type, resolve into the questions as to what topics and problems cover the necessary ground, and best arouse the interest of the student. It will be dangerous to introduce problems involving modern industrial ideas in which the student has had

no proper foundation. Any change in this direction can only consist of the substitution of new names and relations for the old ones, as amperes of current or volts of E. M. F. for yards of cloth. The mathematical processes to be illustrated are not different from what they were a hundred years ago. Good mathematicians were developed on problems of the old type such as Newton, the creator of modern dynamics, Gauss, one of the founders of modern electrical science, Lagrange, Laplace, and many others to whom modern mathematicians must still look up. A moderate use of problems which introduce a few modern scientific and industrial terms may serve the double purpose of adding interest and making the pupil familiar with those terms, but they are no better for mathematical training than problems of the old type.

Of vastly greater importance are changes which tend to narrow the gap between elementary and higher mathematics. Can we give the elementary student the idea that there is something for algebra to do besides find x? Elementary students manifest a decided weakness in their concepts of variation. Problems which deal with variables, such as the relative motions of the hands of a clock, are difficult for elementary students, because they have had no systematic discussion of the principles involved. The concept of independent and dependent variables, or of functions, should be developed by familiar illustrations both geometrically by graphs, and physically by simple motions. The topic of limits should not be introduced into an elementary course without some introductory treatment of variation which is orderly in character.

In conclusion, my advice to a high school which desires to adjust its course to the needs of a technical school is that a review in elementary algebra be given in the senior year, with special attention to clearness, neatness, and accuracy, in the fundamental operations. If there is time, a similar review of elementary arithmetic and geometry, consisting mainly of a course of original exercises, would be useful.

Rose Polytechnic Institute,
Terre Haute, Ind.

WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE CURRICULUM OF THE SECONDARY SCHOOLS? FROM THE POINT OF VIEW OF THE COLLEGE.

By Robert J. Aley.

As educational thought is now adjusting itself, it seems that the time is not far distant when the secondary school curriculum will be made to meet the needs of those who are to profit by it. The college now has less influence than formerly in shaping the curriculum of the secondary school. It will have still less influence in the future unless it studies the problem from the viewpoint of present conditions and makes many needed adjustments in its requirements. I verily believe that the mathematical study which prepares best for college will prove to be of great value to the student who does not go to college.

The college needs students who know how to study. The mathematics of the secondary school, if properly taught, should develop this ability. The definiteness of its problems, and that quality in its nature which compels attention, make mathematics particularly valuable in training how to study. Secondary teachers generally should give more attention to the matter of training in methods of study. Secondary teachers of mathematics should take advantage of the nature of their subject to make this training very specific. The graduate of the secondary school who has learned how to study has a most valuable equipment. If he becomes a college freshman, he is almost certain to acquit himself creditably.

The college needs students who have courage to attack hard problems and persistency enough to make the attack successful. Testimony from many sources leads to the conclusion that the young people entering college to-day are short in this sort of of courage. It is likely that this shortage is in part explained by the general movement of recent years which has tended to make school work easy and pleasant. To some extent, education has been kindergartenized. Our young people have been relieved from doing hard tasks. As a result, we find but few

college freshmen who are willing to give real effort and proper time to the hard problems that come in their work. It seems to me, then, that the college should expect the secondary school to teach and train in such a way that the type of courage needed in the doing of hard things will be developed. The mathematics of the secondary school furnishes a splendid opportunity for such training.

One of the most desirable traits that the college freshman can possess is mental alertness. The student who is awake, whose mind is receptive and who sees quickly the difficulty or explanation, is a joy to his teacher. This quality is developed only by long continued effort. The best time for its development is in the years of secondary education. It seems fair, therefore, for the college to expect the secondary school to train its students so that they may have the mental alertness needed for the best work in college. Mathematics, as well as any other subject, and better than most others, is suited to give this sort of training.

One of the hardest difficulties to overcome in the early years of college life is that of slovenly habits and lack of accuracy. The secondary school ought to develop neatness and accuracy. Mathematics, from its very nature, is well fitted to do these two things. Its results are accurate and its processes are such that neatness is necessary. If the teachers of mathematics in its earlier stages were impressed with the necessity of allowing the subject to have full power in these two particulars, the work of the college teacher would be made much easier and the possible accomplishments of the student would be much greater.

The knowledge of mathematics which the college needs and has a right to expect falls naturally into two divisions. Students who come to college ought to be familiar with the art of computing as it is illustrated in arithmetic and algebra. They ought also to understand thoroughly the scientific principles that lie back of the important truths of arithmetic, algebra and geometry. If the secondary school fails in either of these two particulars, the work in college is greatly handicapped.

I believe that the college has a right to insist that the secondary schools furnish students who know perfectly the ordinary tables of operation and who are able to carry on arithmet-

ical calculations with speed and accuracy. Much time of many freshmen is wasted in the preparation of mathematical lessons because of their inability to do ordinary arithmetical calculations with ease, precision and moderate speed.

The high school as well as the last two years of grammar school ought to give considerable training in mental calculations. Great good would result if mental arithmetic, which once played so important a part in the development of young people, were again restored to a place in the school curriculum. This subject gives power to think straight and quickly.

Both the high school and the latter years of the grammar school should give much training in short and direct methods of calculation. Nothing does more to save time and to develop mental alertness than training of this sort.

In algebra the secondary school should give a complete mastery of the fundamental operations, indices, and equations,—linear, simultaneous and quadratic. One of the important things that the secondary school often fails to do is to teach the student of algebra a proper appreciation of general quantities and type forms. If more emphasis were given to these things, the work of the freshman in college mathematics would be made much easier. College teachers generally agree that freshmen rarely understand the meaning of x, n and a when used in general formulæ. Is it not reasonable to expect the secondary school to fix these matters in the minds of its pupils?

The training in geometry ought to result in an appreciation of the value of logical reasoning. The year and a half devoted to this subject is partially wasted if the student leaves the study without an understanding of what a proof really is. More emphasis upon reason and logic would produce not only better prepared college students, but also men better able to meet every day problems.

The work in geometry should give the student an accurate knowledge of the most fundamental geometrical facts. He ought to know intimately the properties of triangles, rectangles, parallelograms, circles and the various regular solids. This knowledge ought to be on tap in his memory, resting, however, upon a real understanding of the principles. To fix these facts in memory and make them of use, much time should be given to their application to practical problems.

The secondary schools are doing good work and are showing much interest in making their work still better. The colleges should be more sympathetic and more willing to view the problem from the secondary school standpoint. Better cooperation will result in improved products. If the secondary schools will send the colleges students who know how to work, who are open-minded and alert, who are familiar with the fundamentals of arithmetic, algebra and geometry, and who have a curiosity to know, the work in college mathematics will be greatly improved.

University of Maine,
Orono, Me.

NEW BOOKS.

Academic Algebra. By GEORGE WENTWORTH and DAVID EUGENE SMITH. Boston: Ginn and Company. Pp. 442. $1.20.

This book is designed to cover all the topics demanded for entrance to college and all the work required for the boy or girl who is preparing directly for any trade or industry. It is in every sense a new work and is constructed on entirely modern lines.

It begins in a way to arouse at once the interest of the pupil. It shows how algebra grows out of arithmetic; it makes clear at the outset many of the uses and applications of the subject; it correlates with algebra the arithmetic and mensuration that have already been studied.

Higher Algebra. By HERBERT E. HAWKES. Boston: Ginn and Company. Pp. 222. $1.40.

This text, while it does not emphasize the applications of algebra to the detriment of a thorough development of the subject itself, is written in the spirit of applied mathematics and is especially adapted for use in technical schools. It is designed for students who are competent to pass the usual college entrance examinations in elementary algebra.

A concise but illuminating review of the most important features of elementary algebra is followed by a thorough discussion of the quadratic equation and the more advanced topics, including series, which are essential for the student who wishes to make adequate preparation for the calculus.

In view of its particular importance, the chapter on the Theory of Equations is treated with unusual care in the hope that the processes which students often perform in a perfunctory manner will take on life and additional interest.

The Method of Archimedes Recently Discovered by Heiberg. A Supplement to the Works of Archimedes, 1897. Edited by SIR THOMAS L. HEATH. Cambridge: The University Press. Pp. 51. 75 cents.

This pamphlet could not be prepared by a man better equipped for the task. It is transated direct from the Greek, with an introductory note.

It is shown that Democritus and not Eudoxus was the first to assert the truth of the theorem that the volumes of a pyramid and a cone are one third of the volumes of a prism and cylinder respectively which have the same base and equal height.

The method used will be seen to be an ingenious mechanical device for avoiding integration.

Mathematical Wrinkles. By SAMUEL I. JONES. Gunther, Texas: Published by the author. Pp. 321. $1.65.

This book is intended for teachers and private learners and consists of knotty problems; mathematical recreations answers and solutions; short methods; helps; etc. It contains much information and will be a source of much interest to many teachers.

A Text-Book of Mathematics and Mechanics. By CHARLES A. A. CAPITO. Philadelphia: J. B. Lippincott Company. Pp. 398. $4.00 net.

This book has been compiled with the object of assisting English students to prepare for scientific and higher technical examinations. The author takes the sound view that a thorough knowledge of mathematics is a prime necessity for engineers as a basis for their technical work.

In Analytical Geometry the straight line, circle, parabola, ellipse, and hyperbola are all treated separately and then all are shown to belong to one category. In Calculus differentiation and integration are treated separately. Mechanics is treated entirely from a dynamical point of view.

There are many worked out examples and the book is one that will appeal to those who like careful work.

Map Projections. By ARTHUR R. HINKS. Cambridge: The University Press. G. P. Putnam's Sons American representatives. Pp. 126. $1.50

The theory of the conformal representation of one surface upon another is of very great importance in mathematics, but in actual map-making it is of no great advantage. The author of this book therefore departs from the usual course of presenting the general mathematical theory first and then the practical applications, and begins by considering the various projections in common use and discusses the merits and defects of each for a given map. Careful consideration is given to the relations between the various projections; the extent to which they possess the qualifications which a good map projection should possess; the method by which they can be constructed; and the way in which a map so constructed can be used.

Statics. By HORACE LAMB. Cambridge: The University Press. G. P. Putnam's Sons American representatives. Pp. 341. $3.25.

This book presupposes some knowledge of elementary Mechanics on the part of the student, and while the calculus is freely used much use is made of geometrical methods and of those of graphic statics.

Besides the statics of solids five chapters are given on the statics of liquids as well as the elements of the theory of elasticity. It would seem to be a very teachable book.

Matrices and Determinoids. By C. E. CULLIS. Cambridge: The University Press. G. P. Putnam's Sons American representatives. Pp. 430. $7.00 net.

The chief feature of this book is that it deals with rectangular matrices and determinoids as distinguished from square matrices and determinants, the determinoid of a rectangular matrix being related to it just as a determinant is related to a square matrix. The author endeavors to set forth a complete theory of these two subjects, and uses the first volume to give the most fundamental portions of the theory. Two more volumes are promised, the second to give the more advanced portions of the theory, and the third its applications.

This is new ground and the author has done a splendid piece of work and with the publishers deserves much credit.

There is just one criticism which we wish to make and that is no reference is made to work already done. In a book of this kind where the ground is mostly new references cannot be numerous, but to a reader of the book references to the things that have been done before would be very valuable. For instance the work concerning the rank of a matrix is not all new and reference would be useful. Again the author's Q^n_m is the same as Metzler's $\phi(n, m)$ in the *American Journal of Mathematics*, Vol. XXII., No. 1, and in "Netto Combinatorik," § 59.

Nouvelles Tables de Logarithmes. Par Emile Mougin. Published by the author. Roanne (Loire), 1913. 56 pages, with a large table for use in the class-room.

M. Mougin, professor of Mathematics in the Lycée de Roanne, has recently published this ingenious table of logarithms of numbers from 1 to 10,000, of the trigometric functions from 0^λ to 100^γ, natural functions from 0^γ to 100^γ, and the natural and logarithmic functions from 0° to 90°. There is also included a very helpful conversion table. The centesimal table is arranged for tenths of the centesimal minute as well as for 60ths of the sexagesimal minute. The table shows the present tendency away from the old sexagesimal system and illustrates in a successful way the transition period through which we are passing.

Professor Mougin will send a copy of this table gratis to any professor of mathematics who may be disposed to make use of it in his class. The price prepaid is thirty cents.

NOTES AND NEWS.

The twentieth meeting of the Association was called to order by Vice-President Long in Room 107, Thaw Hall, University of Pittsburgh, on Saturday, March 22. After a hearty address of welcome by the Chancellor of the University, Dr. S. B. McCormick, the topic of the morning, "What Mathematical Subjects Should be Included in the Curriculum of the College," was presented by Prof. G. H. Hallett, of the University of Pennsylvania. The same subject was continued by Prof. F. J. Holder, of the University of Pittsburgh, and by Prof. C. C. Guthrie, of the Medical School. The frequent applause showed the appreciation of those present. No resumés of these papers are given as they are to be published. The topic was discussed by Professor Wilson, Professor Eiesland, Professor Hallett, and Dean Metzler.

The report of the Committee on Testing the Results of Geometry Teaching was presented by Mr. Morrison. It is an excellent report and, when available through The Teacher, is worth serious study. The report of the Committee on Affiliation was given by Dean Metzler. It appears now that the project will be carried through and a strong association of eastern mathematics teachers will be formed.

Luncheon was served by the University at the University Club to those present at the meeting. At the close of the luncheon the Chancellor added to his welcome of the morning and the Secretary responded for the Association.

At the opening of the afternoon session the question of "The Comprehensive Examination for Admission to College" was presented in papers read by Prof. A. H. Wilson, of Haverford College, and Principal J. N. Rule, of the Central High School of Pittsburgh. Mr. F. Eugene Seymour, Dr. Metzler, Mr. H. F. Hart, Professor Wilson, and Mr. Morrison spoken in discussion. The general theory of the examination was approved but it was felt that many serious difficulties lay in the way of its proper execution. Among these the chief one seemed to be the difficulty of avoiding cramming.

The program was closed by papers by Professor Eiesland, of the University of West Virginia, Mr. H. F. Hart, of the Montclair High School, Mr. E. H. Koch, Jr., of Pratt Institute, and Mr. F. Eugene Seymour, of the New Jersey State Normal School on "The Comprehensive Examination in the Teaching of Mathematics." Time did not permit of the discussion of these papers. After the adoption of resolutions (printed elsewhere) the meeting adjourned.

Howard F. Hart,
Secretary.

The sixth annual dinner of the Philadelphia Section was held on Saturday evening, February first, in the Palm Room of the Hotel Walton. Covers were laid for seventy-five. The President of the Section, Maurice J. Babb, was toastmaster.

The after-dinner speakers were Dr. William A. Graunlle, President of Pennsylvania College, Gettysburg; Miss Katharine E. Puncheon, of the Philadelphia High School for Girls; Dr. Fletcher Durell, of the Lawrenceville School; Dr. Arthur Quinn, Dean of the College of the University of Pennsylvania.

The evening proved to be one of much pleasure and profit.

The Rochester Section has held two meetings during the current college year. The thirteenth meeting of the Section was held at the Lafayette High School in Buffalo, N. Y., in affiliation with the New York State Teachers' Association on November 26, 1912. About fifty-five teachers attended the meeting. The program was as follows:

"A Definition of Trilinear Co-ordinates," by Wilfred H. Sherk, Lafayette High School, Buffalo.

"Graphical Methods used in the Secondary Schools of Germany," by S. Douglas Killam, the University of Rochester.

"Class Room Treatment of Original Exercises in Plane Geometry," by Harry N. Kenyon, East High School, Rochester.

"Complex Numbers in Secondary Mathematics," by Arthur S. Gale, the University of Rochester.

The annual election of officers resulted as follows: Chairman, William P. Durfee, Hobart College, Geneva; Secretary, Arthur S. Gale, the University of Rochester; Executive Committee, Miss Mary M. Wardwell, Central High School, Buffalo; William

L. Vosburgh, Brockport State Normal School; Harry N. Kenyon, East High School, Rochester.

The fourteenth meeting was held at Hobart College, Geneva, N. Y., on March 15, 1913. There was an attendance of over twenty, and those attending the meeting were entertained by Hobart College at a luncheon served at Hotel Seneca.

Owing to the absence of two of the four speakers announced, one of these absences being due to illness, but two papers were read, namely:

"Some Applications of Trigonometry," by Charles W. Watkeys, the University of Rochester.

"The Results obtained in Germany by the International Commission on the Teaching of Mathematics," by William Betz, East High School, Rochester.

In the forenoon there was an informal discussion of some topics in elementary algebra.

The vacancy in the executive committee caused by the removal to Boston, Mass., of William L. Vosburgh was filled by the election of Miss Eunice M. Pierce, of Lockport.

THE spring meeting of the Association of Mathematical Teachers in New England was held at Massachusetts Institute of Technology, on Saturday, May 3, 1913, with the following program:

"Originals in Geometry," Harry B. Marsh, Technical High School, Springfield; "Certain Classroom Devices in Algebra," A. Harry Wheeler, English High School, Worcester; "A Method of Extracting the Square Root of Numbers," Bertram C. Richardson, English High School, Boston; Discussion. "Efficiency *vs.* the Individual," Prof. L. M. Passano, Massachusetts Institute of Technology; "Mathematics for Technical and Manual Training High Schools," Frederick W. Gentleman, Mechanic Arts High School, Boston; Discussion.

Foreign Speakers to be Heard at the Buffalo Congress.

Among the prominent speakers expected at the Fourth International Congress on School Hygiene at Buffalo the last week in August, according to Secretary General Dr. Thomas A. Storey, of the College of the City of New York, are the following notable delegates from abroad:

Professor H. Griesbach of Mulhausen, Alsace, Germany, founder of these international congresses and president of the first congress held in Nuremburg in 1904.

Dr. L. Dufestel of Paris, France, medical inspector of the Paris schools, and secretary-general of the Third International Congress on School Hygiene.

Dr. James Kerr of London, England, member of the London County Council, for many years an active leader in the field of medical inspection.

Dr. Otto Grennes of Christiania, Norway, organizer of the statistical exhibit, department of education, for the Norway Centenary Exhibition in 1914.

Dr. Ernesto Cacace of Naples, Italy, professor in pediatry, Royal University of Naples.

Professor L. V. Liebermann of Budapest, Hungary, professor hygienic institute, Royal University of Budapest.

Dr. R. Kaz of St. Petersburg, Russia, consulting and school oculist.

Dr. Frederick Lorentz of Berlin, Germany, member of the Society for School Hygiene, Berlin Teachers.

Dr. W. Weichardt of Erlangen, Germany, bacteriological research laboratory.

Dr. J. Bayerthal of Worms, Germany.

Dr. J. Brandau of Cassell, Germany.

Dr. Marx Lobsien of Kiel, Germany.

Dr. Theodore Altschul of Prague, Germany, sanitary inspector.

Dr. D. E. Jessen of Strassburg, Germany, International Commission on Mouth Hygiene.

Dr. Cornelio Budinich of Trieste, Austria, architect.

Dr. Mathilde Gstettner of Vienna, Austria, assistant oculist, Vienna Polyclinic High School teacher, and secretary Austrian School Hygiene Association.

Dr. Leo Burgerstein, professor of the Royal University of Vienna, Austria.

Dr. R. H. Crowley of Bradford, England, Board of Education.

Dr. Cecil Reddie of the New School, Abbottsholme, England.

Dr. M. C. Schuyten of Antwerp, Belgium, professor of the New College, Brussells.

Dr. Albin Lenhardtson of Stockholm, Sweden, director of municipal school, dental clinic.

Dr. Hansen Hakonson of Trondhjem, Norway, head master, school hygiene.

Dr. Daumegon, director of health department, city of Narbone, France.

THE International Commission on the Teaching of Mathematics made its report at the Fifth International Congress of Mathematicians at Cambridge, England, in August. Reports were recieved from eighteen countries, and 150 separate reports were submitted. About fifty more are now in process of preparation, and others are contemplated by various countries. The Central Committee, consisting of Professor Klein (Göttingen), Sir George Greenhill (London), and Professor H. Fehr (Geneva), with Professor David Eugene Smith (New York) added, was continued in office for another period of four years. The American reports have been completed and may be obtained gratis by application to the Bureau of Education, Washington, D. C. It is probable that one or more reports, summarizing the large features of the reports of all other countries, will be prepared by the American Commission during the next four years, and that certain other special lines of work will be undertaken. The Central Committee contemplates holding three international conferences on teaching, the first in France in 1914, the second in Germany in 1915, and the third, with the next Congress, in Stockholm in 1916.

THE *American Mathematical Monthly* for April has the following table of contents:

"Some Things We Wish to Know," by E. R. Hedrick.

"History of the Logarithmic and Exponential Concepts," by Florian Cajori.

"Some Inverse Problems in the Calculus of Variations," by E. J. Miles.

"The Probability Integral Deduced by Means of Developments in Finite Form," by E. L. Dodd.

"Western Meetings of Mathematicians," by H. E. Slaught.

Book Reviews. Problems and Questions. Notes and News.